

# A Hybrid Approach with Artificial Neural Networks, Levenberg-Marquardt and Simulated Annealing Methods for the Solution of Drying Inverse Problems

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## Introduction

The analysis of the simultaneous heat and mass transfer phenomena in porous media is of great interest [1-8] and in most cases the mathematical model used is based on Luikov equations [1,4,5]. More recently, the inverse problem involving the drying phenomena has attracted the attention of several researchers.

In the present work we extend the results of [8] using an ANN [9-10] to generate the initial guess for the LM and another ANN to approximate the gradient needed by LM. Another improvement introduced in the present work is the use of a different choice of the parameters to be estimated, allowing the design of an experiment with higher sensitivity coefficients.

In Fig. 1, adapted from [3], it is represented the drying experiment setup.

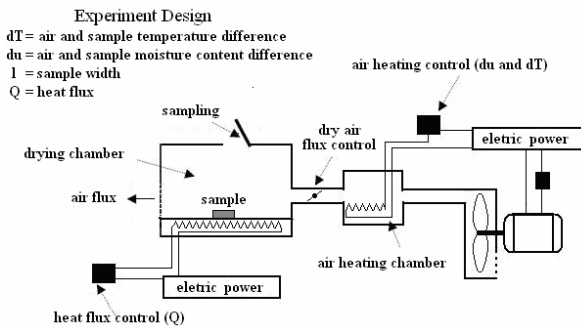


Fig. 1. Drying experiment setup (adapted from [2]).

As many others authors, in previous works we have used measurements of temperature and moisture content in order to estimate the Luikov, Possnov, Kossovich, heat Biot and mass Biot numbers. In the present work we designed an “optimum” experiment and used Artificial Neural Networks, and its hybridization with Levenberg-Marquardt and Simulated Annealing methods, to estimate the Luikov number and others primary properties and property ratios used in Luikov equations.

## Inverse Problem Formulation

The inverse problem is implicitly formulated as a finite dimensional optimization problem [10,11] where one seeks to minimize the squared residues functional

$$S = [\bar{G}_{calc}(\bar{P}) - \bar{G}_{med}]^T W [\bar{G}_{calc}(\bar{P}) - \bar{G}_{med}] \quad (1)$$

where  $\bar{G}_{med}$  is the vector of measurements,  $\bar{G}_{calc}$  is the vector of calculated values,  $\bar{P}$  is the vector of unknowns and  $W$  is the diagonal matrix whose elements are the inverse of the measurement variances.

The inverse problem solution  $\bar{P}^*$  minimizes the norm given by Eq. (1), that is

$$S(\bar{P}^*) = \min_{\bar{P}} S(\bar{P}) \quad (2)$$

## Inverse Problem Solution

Using temperature measurements,  $T$ , taken by sensors located inside the medium and the average value for the moisture content,  $\bar{u}$ , during the experiment, we try to estimate the vector of unknowns  $\bar{P}$  which is a combination of the following variables:  $Lu$  (Luikov number),  $\delta$  (thermogradient coefficient),  $r/c$  (relation between latent heat of evaporation and specific heat of the medium),  $h/k$  (relation between medium and air heat transfer coefficient and thermal conductivity), and  $h_m/k_m$  (relation between medium and air mass transfer coefficient and mass conductivity).

After training, an ANN is able to quickly provide an inverse problem solution. This solution is used as an initial guess for the LM.

A second ANN was trained to calculate the solute concentration, using the information on  $Lu$ ,  $\delta$ ,  $r/c$ ,  $h/k$ ,  $h_m/k_m$  and  $t$ . This ANN was used to provide an

approximation for the Jacobian matrix used in the LM method.

Because of the project space complexity, if convergence is achieved by a deterministic method, the solution can be in fact a local minimum. So, after the LM, the Simulated Annealing (SA) method [7] is used to search for a possibly better solution in the neighborhood of the global minimum. If the SA results in the same

solution given by LM, it is possibly a good approximation for the global minimum.

The results obtained using the LM 1 (gradient approximated by FDM), LM 2 (gradient approximated by ANN), ANN and hybrid combinations, for different values of the standard deviation of temperature and average moisture content experimental errors,  $\sigma_T$  and  $\sigma_{\bar{u}}$ , respectively are shown in Table 1.

**Table 1 – Results obtained using LM 1, LM 2, ANN and hybrid combinations.**

Case	Method	$\sigma_T$	$\sigma_{\bar{u}}$	$Lu$	$\delta$	$r/c$	$h/k$	$h_m/k_m$	Time (s)	$S$ Eq. (1)
1	LM 1 (FDM grad.)	0	0	0.0080	2.00	10.83	34.0	114.0	15	0
2	LM 2 (ANN grad.)	0	0	0.0080	2.00	10.83	34.0	114.0	10	0
3	LM 1 (FDM grad.)	0.08	0.002	0.0076	2.09	10.76	34.1	121.2	16	8881
4	LM 2 (ANN grad.)	0.08	0.002	0.0093	1.71	10.73	34.1	95.7	11	8883
5	ANN	0.08	0.002	0.0083	2.10	10.04	35.0	117.1	1	8893
6	LM 1 (FDM grad.)	0.08	0.002	0.0083	1.92	10.75	35.0	110.0	16	8882
7	LM 2 (ANN grad.)	0.08	0.002	0.0082	1.79	9.89	35.1	114.5	11	8881
8	SA (20.000 evaluations)	0.08	0.002	0.0094	1.58	9.96	35.0	98.2	300	8885
9	ANN-LM-SA SA (2000 evaluations)	0.08	0.002	0.0082	1.97	10.94	33.9	109.2	43	8879

$\sigma_T = 0.08$  and  $\sigma_{\bar{u}} = 0.002$  correspond to errors up to 5% in the experimental data.

The exact values used are:  $Lu = 0.008$ ,  $\delta = 2.0^\circ M/^\circ C$ ,  $r/c = 10.83^\circ C/^\circ M$ ,  $h/k = 34.0m^{-1}$  and  $h_m/k_m = 114.0m^{-1}$ .

While in test cases numbers 1, 2, 3 and 4 the initial guesses are  $Lu = 0.004$ ,  $\delta = 3.0^\circ M/^\circ C$ ,  $r/c = 15.0^\circ C/^\circ M$ ,  $h/k = 50.0m^{-1}$  and  $h_m/k_m = 180.0m^{-1}$ , in test cases numbers 6, 7 and 9 the initial guesses are the estimates obtained with the ANN.

## Conclusions

The hybrid combination ANN-LM-SA was able to produce good solutions for the inverse drying problem.

The use of the ANN to obtain the derivatives in the first steps of the LM method reduced the time required for the solution of the problem.

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